Hyperon and charmed baryon masses from twisted mass Lattice QCD
\((N_f = 2 + 1 + 1 \text{ TMF}, N_f = 2 \text{ TMF} + \text{plus Clover})\)

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C. Alexandrou et al. arXiv:1406.4310

with

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Why we want to calculate baryon masses?

- **easy to calculate**
  first quantities one calculates before proceeding with more complex observables

- **large signal to noise ratio**
  reliable way to study lattice effects

- **significant for on-going experiments**

- **are the experimentally known masses reproduced?**
  safe and reliable predictions for the rest
Lattice evaluation
Wilson twisted mass action for $N_f = 2 + 1 + 1$

- doublet of light quarks: $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$  
  R. Frezzotti et al. arXiv:hep-lat/0306014

Transformation of quark fields:

$$
\begin{align*}
\psi(x) &= \frac{1}{\sqrt{2}} \left( \mathbb{1} + i \tau^3 \gamma_5 \right) \chi(x) \\
\bar{\psi}(x) &= \bar{\chi}(x) \frac{1}{\sqrt{2}} \left( \mathbb{1} + i \tau^3 \gamma_5 \right) \\
\bar{\psi} m \psi &\rightarrow \bar{\chi} i \gamma_5 \tau^3 m \chi
\end{align*}
$$

mass term

$$
S^{(l)}_F = a^4 \sum_x \bar{\chi}(x) \left[ \frac{1}{2} \gamma_\mu (\nabla_\mu + \nabla^{*}_\mu) - \frac{a r}{2} \nabla_\mu \nabla^{*}_\mu + m_{0,l} + i \gamma_5 \tau^3 \mu \right] \chi(x)
$$

- heavy quarks: $\chi_h = \begin{pmatrix} s \\ c \end{pmatrix}$  
  In the sea we use the action:  
  R. Frezzotti et al. arXiv:hep-lat/0311008

$$
S^{(h)}_F = a^4 \sum_x \bar{\chi}_h(x) \left[ \frac{1}{2} \gamma_\mu (\nabla_\mu + \nabla^{*}_\mu) - \frac{a r}{2} \nabla_\mu \nabla^{*}_\mu + m_{0,h} + i \mu_\sigma \gamma_5 \tau^1 + \tau^3 \mu_8 \right] \chi_h(x)
$$

presence of $\tau^1$ introduces mixing of the strange and charm flavors

valence sector: use Osterwalder-Seiler valence heavy quarks $\chi^{(s)} = (s^+, s^-)$, $\chi^{(c)} = (c^+, c^-)$

re-tuning of the strange and charm quark masses required

Wilson TM at maximal twist

- cut-off effects are automatically $O(a)$ improved
- no operator improvement is needed (important for nucleon structure)
Lattice evaluation
Wilson twisted mass action for $N_f = 2$ plus clover

$$S_F^{(l)} = a^4 \sum_x \overline{\chi}(x) \left[ \frac{1}{2} \gamma_\mu (\nabla_\mu + \nabla^*_\mu) - \frac{ar}{2} \nabla_\mu \nabla^*_\mu + m_{0,l} + i\gamma_5 \tau^3_\mu + \frac{i}{4} C_{SW} \sigma^{\mu\nu} F^{\mu\nu}(U) \right] \chi(x)$$

Clover term
- stable simulations
- control $O(a^2)$ effects
- $O(a)$ improvement remains!
- $C_{SW} = 1.57551$

● Total of 10 $N_f = 2 + 1 + 1$ gauge ensembles produced by ETMC
● $N_f = 2$ plus clover ensemble at the physical pion mass


<table>
<thead>
<tr>
<th>$\beta = 1.90$, $a = 0.0936(13)$ fm</th>
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<tbody>
<tr>
<td>$32^3 \times 64$, $L = 3.0$ fm</td>
</tr>
<tr>
<td>$a\mu$</td>
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<tr>
<td>No. of Conf</td>
</tr>
<tr>
<td>$m_\pi$ (GeV)</td>
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<td>$m_\pi L$</td>
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<table>
<thead>
<tr>
<th>$\beta = 1.95$, $a = 0.0823(10)$ fm</th>
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<tbody>
<tr>
<td>$32^3 \times 64$, $L = 2.6$ fm</td>
</tr>
<tr>
<td>$a\mu$</td>
</tr>
<tr>
<td>No. of Conf</td>
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<tr>
<td>$m_\pi$ (GeV)</td>
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<td>$m_\pi L$</td>
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</table>

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<thead>
<tr>
<th>$\beta = 2.10$, $a = 0.0646(7)$ fm</th>
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<tbody>
<tr>
<td>$48^3 \times 96$, $L = 4.5$ fm</td>
</tr>
<tr>
<td>$a\mu$</td>
</tr>
<tr>
<td>No. of Conf</td>
</tr>
<tr>
<td>$m_\pi$ (GeV)</td>
</tr>
<tr>
<td>$m_\pi L$</td>
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</tbody>
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● two lattice volumes
● pion masses from 210-430 MeV → chiral extrapolations
● three values of the lattice spacing → investigation of finite lattice effects
for baryon masses → physical nucleon mass

dedicated high statistics analysis on 17 $N_f = 2 + 1 + 1$ ensembles

use $\text{HBχPT}$ leading one-loop order result

$$m_N = m_N^{(0)} - 4c_1 m_\pi^2 - \frac{3g_A^2}{16\pi f_\pi} m_\pi^3$$

fit simultaneously for $N_f = 2 + 1 + 1$ and $N_f = 2$ plus clover for all $\beta$ values

systematic error due to the chiral extrapolation → use $O(p^4)$ $\text{HBχPT}$ with explicit $\Delta$-degrees of freedom

fitting for each $\beta$ separately yields consistent values - negligible cut-off effects for the nucleon case

light $\sigma$-term for nucleon $\sigma_{\pi N} = 64.9(1.5)(13.2)$ MeV
Effective masses are obtained from two-point correlation functions

\[
C_B^{\pm}(t, \vec{p} = \vec{0}) = \sum_{\text{sink}} \left[ \frac{1}{4} \text{Tr} \left( 1 \mp \gamma_0 \right) \left\langle J_B(x_{\text{sink}}) \bar{J}_B(x_{\text{source}}) \right\rangle \right], \quad t = t_{\text{sink}} - t_{\text{source}}
\]

- Gaussian smearing at source and sink, APE smearing at spatial links
- source position chosen randomly

\[
am_B^{\text{eff}}(t) = \log \left( \frac{C_B(t)}{C_B(t + 1)} \right)
\]
constructed such that they have the quantum numbers of the baryon in interest

4 quark flavors \{ SU(3) subgroups of SU(4) \}

Examples

<table>
<thead>
<tr>
<th></th>
<th>( J = \epsilon_{abc} (u_a^T C \gamma_5 d_b) u_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma^0 ) (uds)</td>
<td>( J = \frac{1}{\sqrt{2}} \epsilon_{abc} [(u_a^T C \gamma_5 s_b) d_c + (d_a^T C \gamma_5 s_b) u_c] )</td>
</tr>
<tr>
<td>( \Xi^+_c ) (usc)</td>
<td>( J = \epsilon_{abc} (u_a^T C \gamma_5 s_b) c_c )</td>
</tr>
<tr>
<td>( \Xi^{*0} ) (uss)</td>
<td>( J_\mu = \epsilon_{abc} (s_a^T C \gamma_\mu u_b) s_c )</td>
</tr>
<tr>
<td>( \Sigma^{*++} ) (uuc)</td>
<td>( J_\mu = \frac{1}{\sqrt{3}} \epsilon_{abc} [(u_a^T C \gamma_\mu u_b) c_c + 2 (c_a^T C \gamma_\mu u_b) u_c] )</td>
</tr>
<tr>
<td>( \Omega^{*0} ) (ssc)</td>
<td>( J_\mu = \epsilon_{abc} (s_a^T C \gamma_\mu c_b) s_c )</td>
</tr>
</tbody>
</table>

20plet of spin-1/2 baryons

\( 20 = 8 \oplus 6 \oplus 3 \oplus 3 \)

20plet of spin-3/2 baryons

\( 20 = 10 \oplus 6 \oplus 3 \oplus 1 \)
Lattice evaluation
Interpolating fields

- incorporation of spin-3/2 and spin-1/2 projectors

\[
P_{3/2}^{\mu\nu} = \delta^{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu, \quad J_{B3/2}^\mu = P_{3/2}^{\mu\nu} J_{\nu}^B \\
P_{1/2}^{\mu\nu} = \delta^{\mu\nu} - P_{3/2}^{\mu\nu}, \quad J_{B1/2}^\mu = P_{1/2}^{\mu\nu} J_{\nu}^B
\]
Tuning of the strange and charm quark mass ($N_f = 2 + 1 + 1$)

- use $\Omega^-$ for strange quark and $\Lambda_c^+$ for charm quark
- fix renormalized strange and charm masses using non-perturbatively determined renormalization constants (N. Carrasco et al. arXiv:1403.4504) in the $\overline{MS}$ scheme at 2 GeV

Strange quark mass tuning

- use a set of strange quark masses to interpolate the mass of $\Omega^-$ to a given value of $m_s^R$ and extrapolate to the continuum and physical pion mass using

$$m_\Omega = m_\Omega^0 - 4c_\Omega^{(1)} m_\pi^2 + da^2$$

- match with physical mass of $\Omega^-$

```
\begin{align*}
\text{Continuum limit} \\
\beta=1.90, L/a=32 \\
\beta=1.95, L/a=32 \\
\beta=2.10, L/a=48 \\
msR = 92.4(6) \text{ MeV}
\end{align*}
```

```
\begin{align*}
\overline{MS}: m_s^R(2 \text{ GeV}) &= 92.4(6)(2.0) \text{ MeV}
\end{align*}
```
Charm quark mass tuning

- follow the same procedure using $\Lambda_c^+$ and fit using

$$m_{\Lambda_c} = m_{\Lambda_c}^0 + c_1 m_{\pi}^2 + c_2 m_{\pi}^3 + d a^2$$

$$m_{\Lambda_c} = 1173.0(2.4) \text{ MeV}$$
Tuning of the strange and charm quark mass ($N_f = 2$ plus clover)

- use $\Omega^-$ for strange quark and $\Lambda_c^+$ for charm quark
- use a set of strange and charm quark masses and interpolate to the physical $\Omega^-$ and $\Lambda_c^+$ mass

> interpolate all the rest hyperons and charmed baryons to the tuned values of $a\mu_s$ and $a\mu_c$
Tuning of the strange and charm quark mass ($N_f = 2$ plus clover)

Interpolation

Hyperons - Charmed baryons

Charmed baryons with strange quarks
Results I: Chiral and continuum extrapolation for $N_f = 2 + 1 + 1$

- fit in the whole pion mass range 210-430 MeV
- include all $\beta$’s
- allow for cut-off effects by including a term $\propto a^2$

Hyperons

- use leading one-loop order continuum HB$\chi$PT
- systematic error due to the chiral extrapolation $\rightarrow$ use $O(p^4)$ HB$\chi$PT
Charmed baryons

- use Ansatz $m_B = m_B^{(0)} + c_1 m_\pi^2 + c_2 m_\pi^3 + d a^2$
- systematic error due to the chiral extrapolation $\rightarrow$ set $c_2 = 0$ and restrict $m_\pi < 300$ MeV

- systematic error due to the tuning for all baryons
- finite-$a$ corrections $\sim 1\% - 9\%$ - cut-off effects are small
- reproduction of experimentally known baryon masses $\rightarrow$ Predictions
Results I: Chiral and continuum extrapolation for $N_f = 2 + 1 + 1$

Cut-off effects

<table>
<thead>
<tr>
<th>Baryon</th>
<th>$d$ (GeV$^3$)</th>
<th>$\beta = 1.90$</th>
<th>$\beta = 1.95$</th>
<th>$\beta = 2.10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Xi_{cc}$</td>
<td>1.08(7)</td>
<td>6.3</td>
<td>5.0</td>
<td>3.1</td>
</tr>
<tr>
<td>$\Xi^*_{cc}$</td>
<td>1.01(10)</td>
<td>5.9</td>
<td>4.6</td>
<td>2.9</td>
</tr>
<tr>
<td>$\Omega_{cc}$</td>
<td>1.20(5)</td>
<td>6.9</td>
<td>5.4</td>
<td>3.4</td>
</tr>
<tr>
<td>$\Omega^*_{cc}$</td>
<td>1.10(7)</td>
<td>6.2</td>
<td>4.9</td>
<td>3.0</td>
</tr>
<tr>
<td>$\Omega_{ccc}$</td>
<td>1.15(5)</td>
<td>5.1</td>
<td>4.1</td>
<td>2.6</td>
</tr>
</tbody>
</table>

$m_\Omega = 1.672(7) + 0.466(4) a^2$

$m_{\Omega_{ccc}} = 4.734(9) + 1.154(10) a^2$
Wilson twisted mass action breaks isospin symmetry explicitly to $O(a^2)$.

It is expected to be zero in the continuum limit.

It manifests itself as mass splitting between baryons belonging to the same isospin multiplets due to lattice artifacts.

$u \leftrightarrow d$ is a symmetry, e.g. $\Delta^{++}(uuu)$, $\Delta^{-}(ddd)$ and $\Delta^{+}(uud)$, $\Delta^{0}(ddu)$ are degenerate.
Results II: Isospin symmetry breaking

- $\Delta$ baryons

- Isospin splitting effects are consistent with zero for all lattice spacings and pion masses
Results II: Isospin symmetry breaking

- **Hyperons**

- Small mass splittings for the spin-1/2 hyperons - decreased as $a \rightarrow 0$

- Splitting is smaller for the $N_f = 2$ plus clover ensemble

- Isospin splitting consistent with zero for spin-3/2 hyperons
Results II: Isospin symmetry breaking

- Charmed baryons

<table>
<thead>
<tr>
<th>Δm (GeV)</th>
<th>Ξcc++ - Ξcc+</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.08</td>
<td></td>
</tr>
<tr>
<td>-0.04</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0.04</td>
<td></td>
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<tr>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>0.12</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>a² (fm²)</th>
<th>Ξcc*++ - Ξcc*+</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>0.008</td>
<td></td>
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<tr>
<td>0.01</td>
<td></td>
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</tbody>
</table>

- very small effects for spin-1/2 charmed baryons
- no isospin symmetry breaking for spin-3/2 charmed baryons
Comparison
Lattice results from other schemes

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$N_f$</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>BMW</td>
<td>$N_f = 2 + 1$</td>
<td>clover fermions S. Durr et al. arXiv:0906.3599</td>
</tr>
<tr>
<td>PACS-CS</td>
<td>$N_f = 2 + 1$</td>
<td>$O(a)$ improved clover fermions A. Aoki et al. arXiv:0807.1661</td>
</tr>
<tr>
<td>LHPC</td>
<td>$N_f = 2 + 1$</td>
<td>domain wall valence quarks on a staggered fermions sea (hybrid) A. Walker-Loud et al. arXiv:0806.4549</td>
</tr>
<tr>
<td>MILC</td>
<td>$N_f = 2 + 1 + 1$</td>
<td>Kogut-Susskind fermion action C.W. Bernard et al. hep/lat 0104002</td>
</tr>
</tbody>
</table>
Octet - Decuplet spectrum

Charm baryons, spin-1/2 spectrum

- ETMC $N_f=2+1+1$
- ETMC $N_f=2$ with CSW
- PACS-CS $N_f=2+1$
- Na et al. $N_f=2+1$
- Briceno et al. $N_f=2+1+1$
- Liu et al. $N_f=2+1$
- G. Bali et al. $N_f=2+1$

• Charm baryons, spin-3/2 spectrum

Ongoing - Future work

- finalize work on baryon spectrum for the $N_f = 2$ plus clover ensemble
- proceed with calculation of other observables ($g_A$, ...)
- new implementation in twisted mass CG inverter to accelerate inversions using deflation leads to large speed-up! (might become even larger...) - Arnoldi algorithm and ARPACK package
- more gauge ensembles from ETMC at the physical pion mass / with $N_f = 2$ plus clover action (?)
Conclusions

- twisted mass formulation with $N_f = 2 + 1 + 1$ flavors provides a good framework to study baryon spectrum
- promising results from $N_f = 2$ plus clover ensemble at the physical pion mass
- physical nucleon mass appropriate to fix lattice spacing when studying baryon masses
- isospin symmetry breaking effects are small and vanish as the continuum limit is approached
- cut-off effects are small and under control
- good agreement with other lattice calculations and with experiment - reliable predictions of the $\Xi_{cc}^*$, $\Omega_{cc}$, $\Omega_{cc}^*$ and $\Omega_{ccc}$ masses

Thank you
Lattice evaluation

Effective mass

\[ m_B^{\text{eff}}(t) = \log \left( \frac{C_B(t)}{C_B(t+1)} \right) = m_B + \log \left( \frac{1 + \sum_{i=1}^{\infty} c_i e^{-\Delta_i t}}{1 + \sum_{i=1}^{\infty} c_i e^{-\Delta_i (t+1)}} \right) \xrightarrow{t \to \infty} m_B, \quad \Delta_i = m_i - m_B \]

\[ m_B^{\text{eff}}(t) \approx m_B^c + \log \left( \frac{1 + c_1 e^{-\Delta_1 t}}{1 + c_1 e^{-\Delta_1 (t+1)}} \right) \]

criterion for plateau selection

\[
\left| \frac{m_B^c - m_B^e}{\frac{1}{2} (m_B^c + m_B^e)} \right| \leq \frac{1}{2} \sigma m_B^e
\]
ETMC $N_f = 2 + 1 + 1$ (this work)
C. Alexandrou et al. (ETMC) arXiv:0910.2419
G. Bali et al. (QCDSF) arXiv:1111.1600
L. Alvarez-Ruso et al. arXiv:1304.0483
M.F.M. Lutz et al. arXiv:1401.7805
S. Durr et al. (BMW) arXiv:1109.4265
R. Horsley et al. (QCDSF-UKQCD) arXiv:1110.4971
ETMC $N_f = 2 + 1 + 1$ (this work)
C. Alexandrou et al. (ETMC) [1]
X.-L. Ren et al. [2]
M.F.M. Lutz et al. [3]
S. Durr et al. (BMW) [4]
R. Horsley et al. (QCDSF-UKQCD) [5]